

SOLUTION OF TRIANGLE

For any triangle ABC, prove that

$$(1) a \sin\left(\frac{B-C}{2}\right) = (b-c) \cos \frac{A}{2}.$$

$$(2) \text{ find } \angle B, \text{ if } a=2 \quad b=3 \text{ and } \sin A = \frac{2}{3}.$$

$$3) \frac{b^2-c^2}{a^2} = \frac{\sin(B-C)}{\sin(B+C)}$$

$$4) a \sin(B-C) + b \sin(C-A) + c \sin(A-B) = 0.$$

$$5) \frac{b^2-c^2}{\cos B + \cos C} + \frac{c^2-a^2}{\cos C + \cos A} + \frac{a^2-b^2}{\cos A + \cos B} = 0$$

$$6) a \cos A + b \cos B + c \cos C = 2b \sin A \sin C.$$

$$7) \text{ find 'b' if } \angle B = 45^\circ, \angle C = 105^\circ \text{ and } a=2.$$

$$8) \frac{a \sin(B-C)}{b^2-c^2} = \frac{c \sin(A-B)}{a^2-b^2}$$

$$9) \frac{a^2 \sin(B-C)}{\sin B + \sin C} + \frac{b^2 \sin(C-A)}{\sin C + \sin A} + \frac{c^2 \sin(A-B)}{\sin A + \sin B} = 0.$$

$$10) \tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cdot \cot\left(\frac{C}{2}\right)$$

$$11) \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$$

$$12) 2\left(b \cos^2 \frac{C}{2} + c \cos^2 \frac{B}{2}\right) = a+b+c$$

$$13) (a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2} = c^2$$

$$14) (a+b+c)(a-b+c) = 3ca \text{ if } \angle B = 60^\circ$$

$$15) \triangle ABC \text{ is isosceles if } \frac{\cos A}{a} = \frac{\cos B}{b}.$$